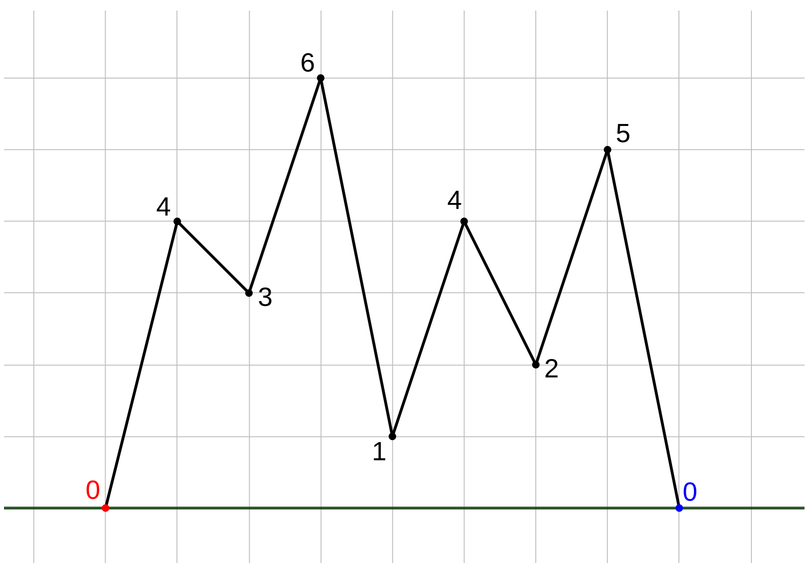
Team

PVMT 2025

All answers will be positive integers, all angles are less than 180°, and all diagrams are drawn to scale. All radical and logarithmic functions are assumed to give positive values. If a problem with a diagram has multiple configurations, refer to the configuration given in the diagram.

Problem 1 ([**2**]). What is the minimum number of 30-60-90 triangles needed to divide a regular hexagon?

Problem 2 ([**2**]). Persons A and B are standing on two opposite sides of a strangely shaped mountain, as shown in the diagram. They want to meet up by traveling on the mountain, but are restricted to constantly stay at the same height - in other words, if A moves up one unit, then B must also move up one unit.



Satisfying this constraint, what is the minimum total vertical motion traveled by A or B (individually) such that the two people meet up?

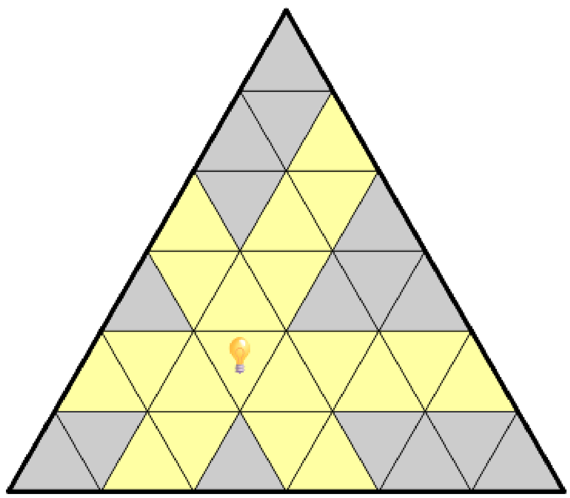
Problem 3 ([**2**]). Consider the following recursive sequence:

$$a_{n-1} = a_0 + a_1 + ... + a_{n-2} + a_n.$$

If $a_0 = 2025, a_1 = 20$, find a_{25}.

Problem 4 ([**3**])**.** Consider a 5×5 grid of vertices where a loop must travel orthogonally between vertices and may not visit a vertex more than once. A loop is "Castled" if it turns 90 degrees on all vertices it passes through. How many "Castled" loops can be drawn?

Problem 5 ([**3**])**.** Ricky wants to set an Akari in a triangular grid with side length 13. A light bulb placed in a cell illuminates all cells which share a row in any direction. No light bulb may illuminate another light bulb. What is the maximum amount of light bulbs Ricky can place in the grid? Example of a light bulb in a size 6 grid:



Problem 6 ([**3**])**.** Ricky has a 11×11 grid and places some number of 2×2 blocks on it. Logan tries to place another 2×2 block on the grid, but surprisingly, they discover that they cannot place the block on the grid without hitting an already placed block. What is the minimum amount of blocks Ricky could have placed?

Problem 7 ([**3**])**.** Find the sum of all roots of the function $\frac{1}{2025}x^4 - \frac{16}{675}x^3 + \frac{1}{15}x^2 + 2x^2\cos(x) - 96x\cos(x) + 270\cos(x)$.

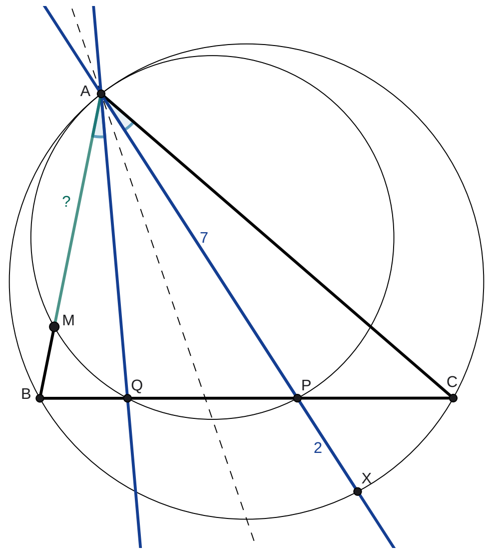
Problem 8 ([**3**])**.** The 2025 PVMT participants are divided into $n > 1$ rooms for the award ceremony. Each participant is associated with a nonnegative score. A participant from the first room moves to the second room, and the average scores in both of the rooms increase by 1 point. A participant from the second room then moves to the next room, and the average scores in both of the rooms again increase by 1 point. This continues until a participant from the nth room moves to the first room, at which point the average score in the nth room increases by 1, and the average score in the first room decreases by k. For how many integer values of k is this possible?

Problem 9 ([**4**])**.** Let $\triangle ABC$ be a triangle with $\overline{AB} = 13, \overline{BC} = 14, \overline{CA} = 15$. Extend segments AB, BC, CA past A, B, C, to lines. Suppose we construct some pair of points P, Q on lines AB and AC respectively such that $PQ = 21$, and $PC \parallel QB$. Construct another parallel line to PC, QB through A that intersects BC at R. Find the distance from R to PQ.

Problem 10 ([**4**])**.** Alice and Bob, who are perfect logicians, both choose an integer between 1 and 100, inclusive. They then figure out each other's numbers with probabilities.
Alice: My number has at least a 50% chance of being greater than yours.
Bob: So, my number has at least a 50% chance of being less than yours.
Alice: There is a chance that they're equal.
Bob: Now my number has at least a 50% chance of being greater than yours.
Alice: Then my number is less and there is at least a 50% chance that our numbers share a factor.
Bob: If your number is one of two options, then they do.
Alice: Now there is a nonzero but less than 50% chance that our numbers share a factor.
Bob: Then they do share a factor!
What is the sum of their numbers?

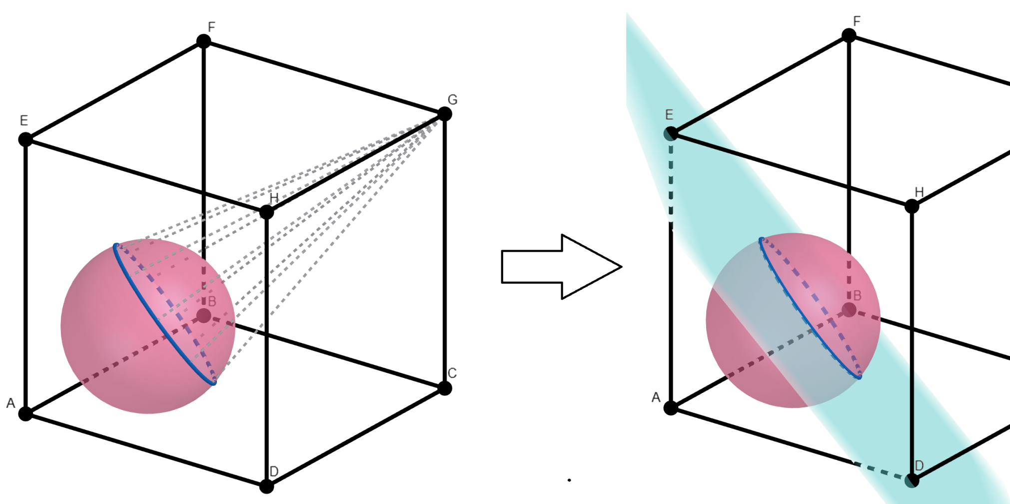
Problem 11 ([**4**])**.** Ricky still has a 11×11 grid, but this time, instead of blocks, he decides to place some number of 1×2 dominoes on it. Logan tries to place another 2×2 block on the grid, but surprisingly, they discover that they cannot place the block on the grid without hitting an already placed block. What is the minimum amount of dominoes Ricky could have placed?

Problem 12 ([**5**])**.** In triangle $\triangle ABC$ with $AB = 6, AC = 9$, let P and Q be two points on line BC such that $\angle BAP = \angle CAQ$. Let AP intersect the circumcircle of $\triangle ABC$ at X, and let the circumcircle of $\triangle APQ$ intersect AB distinct from A at M. If $AX = 9$, $PX = 2$, find length AM. Your answer should be in the form of a fully reduced fraction $\frac{m}{n}$, submit $m + n$.



Problem 13 ([**5**])**.** Define the *polar plane* of a point P around sphere $\mathcal{S}$ as the plane formed by drawing the cone-shaped set of tangent lines passing through P to $\mathcal{S}$, and then drawing a plane through the circle of tangency points on $\mathcal{S}$.

In the cube $ABCDEFGH$, with side length 24, labeled as shown in the diagram, such that A and G are opposite vertices, we draw a sphere $\mathcal{S}$ tangent to the faces $AEDH, AEBF, ABDC$, such that the polar plane of G in $\mathcal{S}$ passes through B, E, D. Find the radius of this sphere. Your answer should be in the form $a - b\sqrt{c}$, submit $a + b + c$.



Problem 14 ([**5**])**.** You have three complex numbers x, y, z such that

$$x + y + z = 1,\ xy + yz + xz = -2,\ xyz = 4.$$

Evaluate $(x^2 - x + 1)(y^2 - y + 1)(z^2 - z + 1)$.

Problem 15 ([**6**])**.** Find the number of integer solutions to

$$\frac{2}{5}xy + \frac{x^2}{5} + \frac{2y^2}{5} - x = 0.$$

Problem 16 ([**6**])**.** Let the cubic curve $\mathcal{K} : y = x^3 + 3x^2 - x + 2$ be defined in the real plane. Let the circle $\mathcal{C} : (x+1)^2 + (y-4)^2 = 5$ intersect the curve at the six points $P_1, P_2 \dots P_6$. Let S_x, S_y be the sum of x and y coordinates of all of the intersection points. Find $-S_x \cdot S_y$.

Problem 17 ([**6**])**.** An infinite line of lily pads has the property that the ith lily pad grants 3^{i-1} gold upon landing on it. Alyssa the frog starts at the first lily pad with 1 gold. Every minute she jumps forward either one or two lily pads, such that if she lands on the ith lily pad for $i \geq 3$, there is a $\frac{1}{3}$ probability she came from the $i-1$th lily pad and $\frac{2}{3}$ probability she came from the $i-2$th lily pad. Given that she eventually lands on the 45th lily pad, the expected number of coins she has upon landing there is $\frac{m}{n}$ for some relatively prime positive integers m and n. Determine the number of factors of n.

Problem 18 ([**6**])**.** Let $\triangle ABC$ be a triangle with circumradius 25. Let I be the incenter of $\triangle ABC$. Let the incircle touch line BC at D, and let AD intersect the incircle again at T. Draw a line ℓ at T tangent to the incircle of $\triangle ABC$ that intersects the circumcircle of $\triangle ABC$ at two points P, Q.

We know that $PQ = 14$. Let M be the minor arc midpoint of PQ. Compute MI^2.

Problem 19 ([**6**])**.** How many ways are there to shade a 4x4 grid so that no 2 shaded cells are orthogonally adjacent?

Problem 20 (Tiebreaker)**.** Spunk the spider is going delusional with hunger. He sees a fly at each of the 5th roots of unity on his unit web in the complex plane. The probability that the fly is in Spunk's imagination is the absolute value of the imaginary component of its position; iff it is not imaginary, it provides 2 calories. Spunk travels counterclockwise, starting at $1 + 0i$ with 2 calories, and eats every fly he comes across; it takes 1 calorie to travel between consecutive roots of unity. If he makes it back to $1 + 0i$ he stops. If he hits 0 calories where a fly is not immediately available, he dies. What is the probability he survives? Estimate the answer as a decimal.