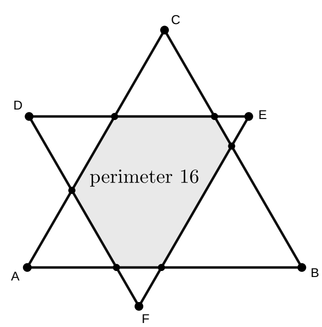
Geometry

PVMT 2025

All answers will be positive integers, all angles are less than 180°, and all diagrams are drawn to scale. All radical and logarithmic functions are assumed to give positive values. If a problem with a diagram has multiple configurations, refer to the configuration given in the diagram.

Problem 1 ([**2**]). Let $ABCD$ be a rectangle with side length $AB = 8$, $BC = 10$. Let the two lines AC and BD intersect at X. What is the area of triangle ABX?

Problem 2 ([**2**]). Let two equilateral triangles $\triangle ABC$ and $\triangle DEF$ point in opposite directions, with their respective sides parallel, such that $AB \parallel DE, EF \parallel BC, FD \parallel AC$. Let these two equilateral triangles intersect in six points that form a convex hexagon with perimeter 16. What is the perimeter of the whole twelve-sided (concave) shape?



Problem 3 ([**2**]). In rectangle $ABCD$ with $AB = 12$, $BC = 8$, we draw two perpendicular line segments PQ and RS spanning the rectangle, such that P lies on AB, Q lies on CD, R lies on AD, and S lies on BC. If $PQ = 9$, find $6 \cdot RS$.

Problem 4 ([**3**]). Triangle PHS has length $PH = 13$, $HS = 14$, and $SP = 15$. Let T and M be the midpoints of PH and HS respectively. Place V on SP such that $PV = 5$. What is the area of quadrilateral $PVMT$?

Problem 5 ([**3**]). An equilateral triangle is inscribed in a circle, which is inscribed in a square. The ratio of the square's area to the triangle's area can be represented as $\frac{a\sqrt{c}}{b}$ fully simplified. Find $a \cdot b \cdot c$.

Problem 6 ([**4**]). Define two parallel lines l_1 and l_2 that are distance 4 apart and points A, B, and C on l_1 in that order where $AB = 4$ and $BC = 6$. Define D, E, and

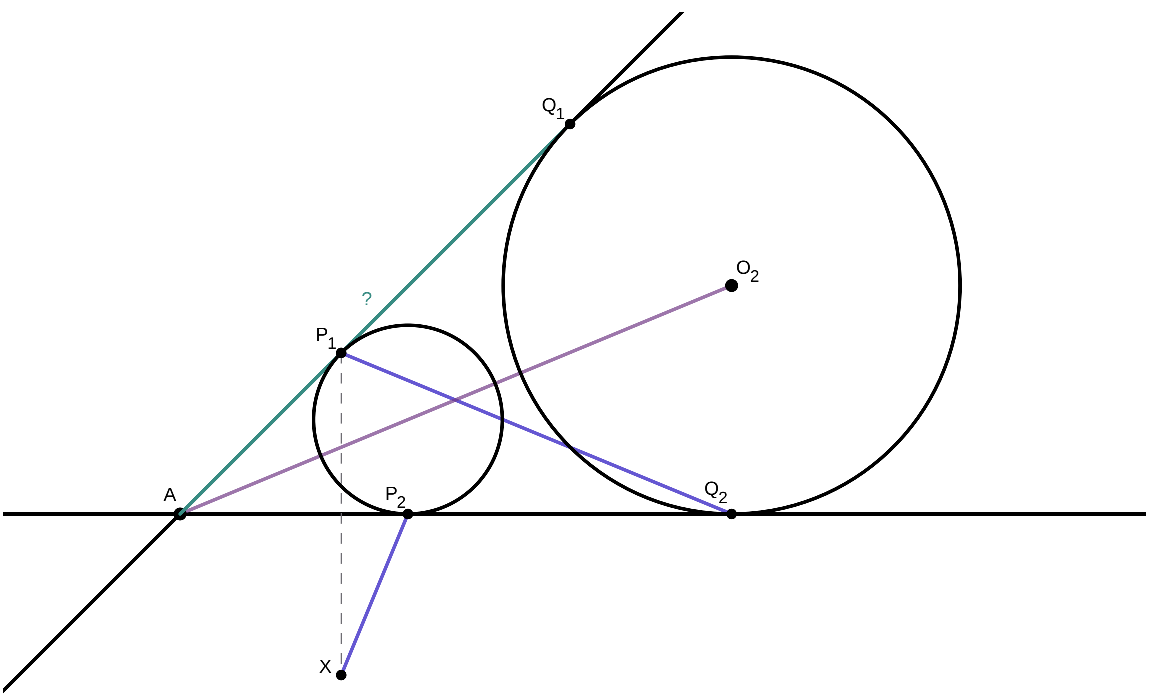
F on line l_2 in that order such that BE is perpendicular to l_2, $DE = 2$, and $EF = 10$. If AF and BE intersect at X, and AF and CD intersect at Y, find $AX \cdot CY$. Your answer should be in the form of $\frac{a\sqrt{c}}{b}$ when fully reduced, submit $a + b + c$.

Problem 7 ([4])**.** Points A, B, and D form an acute triangle and M is the midpoint of AB. Let C be the intersection of the angle bisector of A with the line going through M parallel to AD, and let E be the intersection of AB and CD. If $\angle ADC = 2\angle ACM$, $AM = 5$, $BC = \sqrt{33}$, and $AD = 6$, find BE.

Problem 8 ([4])**.** Let A be a point and draw lines ℓ_1, ℓ_2 from A such that the angle between ℓ_1, ℓ_2 is $45°$, and that ℓ_1 is anticlockwise of ℓ_2 wrt. A. Define two circles O_1, O_2 within the acute angle formed by ℓ_1, ℓ_2 that are tangent to both lines. Let O_1 be tangent to ℓ_1, ℓ_2 at P_1, P_2 and let O_2 be tangent to ℓ_1, ℓ_2 at Q_1, Q_2. Let X be the reflection of P_1 over ℓ_2. Suppose

$$\overline{P_1Q_2} + \overline{XP_2} = \overline{AO_2}.$$

If $AP_2 = 2$, find the length AQ_1. Your answer should be in the fully reduced form $x\sqrt{y} + z$, submit $x + y + z$.

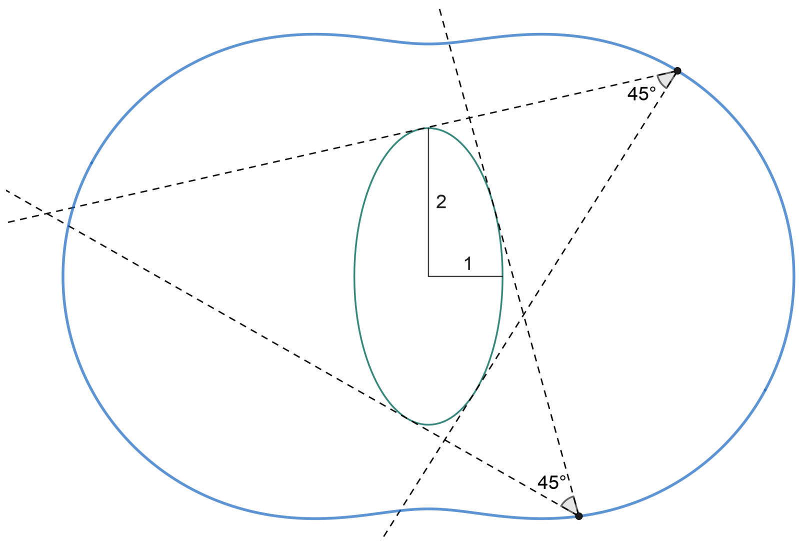


Problem 9 ([5])**.** Alyssa draws n lines in 3D space, with no pair of lines intersecting. She finds that she can actually draw two additional distinct lines, not necessarily intersecting, each of which intersects every of the previously drawn n lines exactly once. Further, she cannot draw another distinct line that intersects all of the n lines. What is the minimum value of n?

Problem 10 ([**5**])**.** A stick of length 6 is randomly oriented in 3D space. The probability that it can pass through a fixed circular hoop of diameter 3 using translation only can be written in the form $\frac{p-\sqrt{q}}{r}$. Find pqr.

Problem 11 (Tiebreaker)**.** Define the $n°$*-isoptic curve* of a (smooth and bounded) shape as the set of points P in the plane such that P "sees" the shape at an angle of $n°$ degrees (i.e. the tangent lines from P to the shape are maximally separated by $n°$ degrees).

Estimate the area enclosed by the 45°-isoptic curve of an ellipse with semi-minor axis 1 and semi-major axis 2.



(The specific curve given in the case of a $n°$*-isoptic curve* of an ellipse is generally known as a *Cassini oval*. Surprisingly, when $n° = 90°$, the curve is a circle known as the *director circle* of the ellipse.)