Combinatorics

PVMT 2025

All answers will be positive integers, all angles are less than 180°, and all diagrams are drawn to scale. All radical and logarithmic functions are assumed to give positive values. If a problem with a diagram has multiple configurations, refer to the configuration given in the diagram.

Problem 1 ([**2**])**.** During a field trip, Mr. Khetarpal must split Alex, Alyssa, Anna, Caleb, Ricky, and Soham into two groups, where one group goes to the farm and the other goes to McDonalds. If each group must have at least 2 people (not including Mr. Khetarpal), Soham wants to go to McDonalds, and Alex wants to go to the farm, how many ways are there to make the groups such that everyone is happy?

Problem 2 ([**2**])**.** How many subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ satisfy the condition that no two elements have an absolute difference of 2, 3, or 4?

Problem 3 ([**2**])**.** How many integers between 10 and 2025 have their digits in strictly ascending order?

Problem 4 ([**3**])**.** How many ways are there to shade a 6×6 grid so that no shaded cell has an unshaded cell directly below or to the left of it?

Problem 5 ([**3**])**.** Alex has a chocolate bar of length n, and wants to share some with Alyssa, but is unsure how much Alyssa wants. Specifically, the bar must break into pieces such that for every integer from 1 to n inclusive, there must exist a subset of pieces whose lengths sum up to that integer. However, each break divides the bar into 3 pieces: the piece to the left of the break, a piece of length 1, and the piece to the right of the break. If Alex only needs to break the bar 4 times, what is the maximum possible length n?

Problem 6 ([**4**])**.** Mr. Khetarpal has a drawer with 9 socks, 3 socks each of 3 different colors, and 1 misplaced glove. He randomly takes items from his drawer until he accumulates a pair of socks of the same color. The expected number of items Mr. Khetarpal takes can be written as m/n where m and n are relatively prime integers. Find $m + n$.

Problem 7 ([**4**])**.** Anna has 3 boxes, currently ordered so that only the second box contains candy. Each second, Anna swaps boxes 1 and 2 or boxes 2 and 3 with equal probability. Let m/n be the probability that the first box contains candy after 10 seconds where m and n are relatively prime integers. Find $m + n$.

Problem 8 ([**4**])**.** Consider the set of rectangles with relatively prime integer side lengths. For a given rectangle, let q be the minimum number of squares required to tile the entire rectangle without overlap. Find the area of the largest rectangle with $q = 8$.

Problem 9 ([**5**])**.** Consider a unit square $ABCD$ where the four sides are walls. A small ball starts at point A and travels in a straight line until it hits wall CD at point P not equal to C or D and is reflected. The ball stops once it has hit a corner. How many possible distinct points P are there such that the ball takes less than 25 reflections to end up at point C?

Problem 10 ([**5**])**.** Initially, Alex is the only member of the 4-member Gaussip club who knows a secret. Each day, everyone who knows the secret randomly selects one of the 3 other members of the Gaussip club and tells them the secret. The expected number of days before all 4 members know the secret is m/n where m and n are relatively prime integers. Find $m + n$.

Problem 11 (Tiebreaker)**.** How many solutions are there to the following Sudoku? Sudoku rules: Place the digits from 1-9 once in every row, column, and 3x3 box.

1			6					4
	2			7			9	
		3						
			4			1		
3				5				7
		2			6			
						7		
	1			2			8	
8					5			9