PVMT 2024: Team Round

Problem 1

Evaluate

$$(\frac{\sqrt{2}+2}{1+\sqrt{2}})^2(1^{\log_{12}21})$$

Problem 2

A random permutation of the numbers 2,0,2,4 is chosen and assigned, in the order they are permuted, to the variables P, V, M, and T, respectively. The probability that the value of the expression $\frac{\frac{P}{V}}{\frac{M}{T}}$ is defined is $\frac{p}{q}$. What is $p+q$?

Problem 3

Let PQR be a 30-60-90 triangle with hypotenuse $PQ = 2$ and $\angle Q = 60^\circ$. Let circle Ω be tangent to PR and RQ with a diameter AB where A and B are on segment PQ and A is closer to P than Q. $BQ = \frac{a\sqrt{b}-c}{d}$ for positive integers a,b,c,d with b squarefree. What is $a+b+c+d$?

Problem 4

Let a,b,c,d,e be distinct integers from -2 to 2. If $a+b^2+c^3+d^4+e^5 = 10$, what is the value of $a+2b+c^2+2d+e$?

Problem 5

There is a class of students at Poolesville such that exactly 400 different pairs of those students are friends and it is possible to divide the students into 3 groups such that each group contains no friendships. What is the minimum possible size of this class?

Problem 6

How many ways are there to arrange the string "122345" such that no number goes into its original position (neither of the 2's can go into positions 2 or 3)?

Problem 7

Let $n = 100! + 101$. Find the sum of the last 20 digits of n^7.

Problem 8

In a circular room with radius 1, the entire wall is covered with mirrors. A person is standing $\frac{9}{10}$ units from the center of the room and shines a laser such that the beam forms a regular hexagonal path. Let the distance the beam travels from the person to where it first bounces off a wall be d, the absolute difference between the two possible values of d is $\frac{\sqrt{a}}{b}$. What is ab?

Problem 9

At the center of regular hexagon $ABCDEF$ lies equilateral triangle PQR such that $BCQP$, $DERQ$, and $FAPR$ are rectangles. A triangle is randomly chosen in the hexagon, the probability that the triangle does not intersect any of the rectangles is $\frac{m}{n}$. What is $m+n$?

Problem 10

A robot is standing at the origin of a Cartesian plane. It can only move right or up by one unit, and the goal is to reach the point $(7,7)$. However, there are some blocked points in the plane where the robot cannot step on. The blocked points are on $(1,2)$, and $(5,6)$. How many different paths can the robot take to reach its goal at point $(7,7)$?

Problem 11

Consider the following curve,

$$(x+y)((x+y)^2-x-y-3)=2-4xy$$

The closest this curve gets to the origin squared is $\frac{a}{b}$. What is $a+b$?

Problem 12

Consider 5 regular polygons from the set of triangles, squares, and pentagons all with equal side length. These polygons are arranged on a plane without any overlapping areas, such that each newly placed polygon shares at least one edge with an already placed polygon. In the configuration that contains the minimum possible distance between 2 vertices, how many total vertices are there? Overlapping vertices are counted as a single vertex.

Problem 13

The value of

$$\sum_{n=1}^{\infty} \frac{n\sin(n)}{2^n}$$

can be represented as $\frac{n\sin(1)}{([2-\cos(1)]^2+\sin^2(1))^2}$. What is n?

Problem 14

In triangle ABC, let H be the orthocenter and O be the circumcenter. The circumradius of ABC is 7. Given that $AO = AH$ and OH is 2, the distance between A and the minor arc BC can be expressed as $a\sqrt{b}$ for integers a and b, where b is squarefree. What is $a \cdot b$?

Problem 15

Sumedh is standing at the point $(\frac{1}{3}, \frac{1}{4})$ in a room with walls on the unit circle. Sumedh walks around until he hits the wall. If he hits the wall at angle θ, then he scores $\cos(2\theta)$ points. Sumedh's expected score is $\frac{p}{q}$, what is $p+q$?