PVMT 2022: High School Division Geometry Round

Problem 1

Alice is standing 5 meters away from Bob. Bob is standing 20 meters away from Charlie. In meters, what is the minimum possible distance between Alice and Charlie?

Problem 2

Let S be the set of points in the coordinate plane that are a distance 11 away from the origin. Let T be the set of points that are a distance of at most 3 from a point in S. If the area of T can be expressed as $n\pi$, find n.

Problem 3

In $\triangle ABC$, $AB = 5$, $BC = 12$, and $AC = 13$. A square with side length s is constructed such that one of its sides lies on AC, and each of its vertices lies on a side of $\triangle ABC$. Given that s can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers, find $m+n$.

Problem 4

Take the ellipse with equation

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

It is then rotated 30 degrees clockwise and a rectangle is circumscribed around it. What is the minimum possible area of this rectangle?

Problem 5

What is the volume of the region enclosed by the graphs of

$$x^{2022} + y^{2022} + z^{2022} = 1, (x - \frac{1}{2})^{2022} + y^{2022} + (z+1)^{2022} = 2$$

Round your answer to the nearest integer.

Problem 6

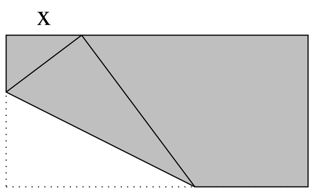
Points A, B, C, and D are in a Cartesian plane such that $A = (2048, 2058)$, $B = (2018, 2018)$, $C = (2036, 2046)$, and $D = (2060, 2039)$. Given that point P is in the same plane, the minimum possible value of $PA + PB + PC + PD$ is closest to which integer?

Problem 7

Let $\triangle ABC$ be an equilateral triangle, and let P be a point inside $\triangle ABC$ such that the distances from P to AB, AC, and BC are 2, 4, and 7, respectively. The area of $\triangle ABC$ can be written as $\frac{m}{\sqrt{n}}$, where m is a positive integer and n is not divisible by the square of any prime. What is $m+n$?

Problem 8

A 2 by 1 strip of paper is folded so that a corner meets a point a distance x along on the opposite edge (see figure). Let x be the value such that the area of the resulting shape is maximized. Suppose x^4 can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.



Problem 9

A point P is placed in equilateral triangle $\triangle ABC$ such that $\angle APB = 100$, $\angle BPC = 120$, and $\angle APC = 140$. A triangle $\triangle XYZ$ is constructed such that $AP = XY$, $BP = XZ$, and $CP = YZ$. Find the measure of the largest angle of $\triangle XYZ$.

Problem 10

In triangle $\triangle ABC$, we have $AB = 13$, $BC = 14$, and $AC = 15$. Let I be the incircle of $\triangle ABC$, and let D, E, F be the tangency points of the incircle to sides BC, AC, AB, respectively. Let X be the foot of the perpendicular from D to EF, let Y be the midpoint of DX, and let K be the orthocenter of $\triangle BIC$. Finally, let Z be the intersection between YK and EF. The value of $(ZE)(ZF)$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.