PVMT 2022: Middle School Division Combinatorics Round

Problem 1

How many ways are there to arrange 5 different people in a line?

Problem 2

In a bag, there are two green marbles and five blue marbles. Bob chooses two marbles from the bag at random, without replacement. If the probability that he chooses two marbles of different colors is $\frac{p}{q}$ with $gcd(p,q) = 1$, find $p+q$.

Problem 3

Bob has a jar that can hold up to one gallon of water. Bob first pours out half the water from the jar, and then pours out a third of the remaining water, and then pours out a fourth of the remaining water... and so on. Starting from a full jar, after how many pours will Bob have $\frac{1}{2022}$ gallons of water left?

Problem 4

Franklin is stuck inside of the world of Kemsuk! He is currently residing on the origin and in order to escape, he must cross a 6×8 grid and reach the land of Fisbest located at the coordinates $(6,8)$.

At any point, Franklin can only run 1 unit up or 1 unit to the right. How many paths are there from Kemsuk to Fisbest?

Problem 5

In a lottery, a ticket consists of 5 distinct, unordered digits between 0 and 9. What is the minimum number of lottery tickets that needs to be created such that it can be guaranteed that at least 6 lottery tickets are identical?

Problem 6

You flip a coin and roll a die, if the number on the die is odd and the coin lands tails or the number on the die is even and the coin lands heads, then you stop, otherwise you repeat. What is the expected number of times you do this before you stop?

Problem 7

Oryan, a student at Fisbad, throws 2 balls into containers labeled $1,2,\cdots n$. If the probability that the ball lands in container k is 2^{-k}, the probability that the balls land in the same container can be represented as $\frac{p}{q}$ where $\gcd(p,q) = 1$ Find $p+q$.

Problem 8

At a school:

1. Students can take either French or Spanish

2. Students cannot take both French and Spanish, but can take neither
3. 250 students take French
4. 300 students take Spanish
5. 300 students take history
6. Every student takes at least one of : French, History, and Spanish

What is the difference between the maximum and minimum number of students that could be at the school?

Problem 9

A bug starts at $(0,0)$ on a Cartesian plane. Each move, the bug can move to any lattice point exactly $\sqrt{5}$ units away (essentially like a knight in chess), as long as it stays inside or on the square with vertices $(0,0)$, $(0,2)$, $(2,0)$, and $(2,2)$. Let A be the probability that after 300 moves, the bug is back on $(0,0)$; also, let B be the probability that after 300 moves, the bug is on $(2,2)$. The value of $A-B$ can be written as $\frac{1}{n}$ for a positive integer n. Find the remainder when n is divided by 100.

Problem 10

In the equation $x = 4?4?4?4?4?4?4?4?4$, each question mark is replaced by one of the four operators ($+$, $-$, $\times$ or $\div$). If standard order of operations is followed, then the expected value of x can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find the number of positive factors of n.