PVMT 2022: High School Division Combinatorics Round

Problem 1

How many ways are there to arrange 5 different people in a line?

Problem 2

Franklin is stuck inside of the world of Kemsuk! He is currently residing on the origin and in order to escape, he must cross a 6×8 grid and reach the land of Fisbest located at the coordinates $(6, 8)$.

At any point, Franklin can only run 1 unit up or 1 unit to the right. How many paths are there from Kemsuk to Fisbest?

Problem 3

Mr. Gerard is playing a betting game with his favorite students during lunch, with weighted coin that lands on heads 20% of the time. In the game, the weighted coin is flipped n times and Mr. Gerard bets that the coin will land on heads exactly 3 times. Given that the probability of the coin landing on heads exactly 3 times in n flips is the same as the probability that the coin lands on heads exactly 4 times in n flips, find the value of n.

Problem 4

A city is holding a lottery. In this lottery, a person can pick tickets that consist of 5 unique digits between 0 and 9. When the winning ticket is drawn, the holder of a ticket that has all 5 of winning digits gets $1036 (the order of the digits doesn't matter). However, the lottery also includes smaller prizes for when 3 or 4 digits match. When a ticket has 4 matching digits, the holder wins $100, and when a ticket has 3 matching digits, the holder wins $10.

What is the minimum value that the cost of the tickets need to be greater than such that the city can be expected to profit off of the lottery?

Problem 5

Aidan has 2022 boxes numbered from 1 to 2022 and an infinite number of balls. He can throw a ball into any box which is a prime number less than 20: 2, 3, 5, 7, ..., 19. If three balls occupy the same box, then they are all removed and a new ball is put into the next. Aidan wants a ball in the 2022nd box. Let T be the ratio of the maximum possible number of throws to the minimum possible number of throws that accomplishes this. Given that T can be expressed as p^q for positive integers p, q, find pq.

Problem 6

You are playing a game with a friend. Your probability of winning one round is $\frac{1}{10}$. But your friend is nice and says that if you lose the first round, you can play twice more, and if you win the second and third, you win the entire game. In fact, if you lose the first 2 games, then you can play three more, and if you win all three of those rounds, you win the game. This rule applies for any first n rounds you lose, if you win the next n+1 you win the entire game.

What is the probability of you winning? Your answer will be a fraction written simplified as $\frac{p}{q}$, give us p+q.

Problem 7

A new gacha game, *Shingen Pactim*, just came out and it's all the rage. In Shingen Pactim, each destinyä player spends allows them a chance at obtaining a featured 5* character. The chance is normally 20%, but starting from the 75th destiny spent, the chance increases linearly until it hits 100% for the 90th roll. Timmy really wants the newest 5* character to round out his team composition, but he doesn't have enough destinies for 90 rolls. He's already spent 73 destinies with no luck and only has 3 destinies left.

If the probability that Timmy will get the character within those 3 wishes can be expressed as $\frac{m}{n}$ where m and n are two relatively prime whole numbers, what is $m+n$?

Problem 8

On some mxm grid of squares, an n-L is a move where, from your starting square, move n squares in one direction, then another 1 to the left or right in that direction of motion. This is an elongated L move a knight makes in chess.

Let f(n) be the least m such that for all M≥m, starting in a corner, you can reach every square of an MxM grid using only n-L moves. If there is no such value, f(n)=0.

What is f(2022)?

Problem 9

A bug starts at $(0,0)$ on a Cartesian plane. Each move, the bug can move to any lattice point exactly $\sqrt{5}$ units away (essentially like a knight in chess), as long as it stays inside or on the square with vertices $(0,0)$, $(0,2)$, $(2,0)$, and $(2,2)$. Let A be the probability that after 300 moves, the bug is back on $(0,0)$; also, let B be the probability that after 300 moves, the bug is on $(2,2)$. The value of $A-B$ can be written as $\frac{1}{n}$ for a positive integer n. Find the remainder when n is divided by 100.

Problem 10

In the equation $x = 4?4?4?4?4?4?4?4?4$, each question mark is replaced by one of the four operators ($+$, $-$, $\times$ or $\div$). If standard order of operations is followed, then the expected value of x can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find the number of positive factors of n.