PVMT 2022: Middle School Division Algebra/Number Theory Round

Problem 1

Find the number of positive integers less than or equal 100 that are divisible by neither 2 nor 5.

Problem 2

How many digits are in the number $8^5 \times 5^{10}$?

Problem 3

In the land of Kemgod, Franklin can use each dollar to buy either 5 grams of chemical A or 6 grams of chemical B. Let a number n be *purchasable* if Franklin can spend some amount of money to buy n total grams of chemicals. If Franklin has infinite money, what is the largest number that is not *purchasable*?

Problem 4

Consider the polynomial $f(x) = x^2 + 4x - 12$. Find the absolute value of the sum of all x such that $f(f(x)) = 0$.

Problem 5

The polynomial $x^5 - ax^3 - bx^2 + cx + d$ has five distinct roots. Four of these roots are $-7, -4, -3$, and 9. Find the value of d.

Problem 6

Neel has a third-degree polynomial, P. He then found $P(x), P(x+1), P(x+2), P(x+3), P(x+4)$ for some fixed x. However, he then realizes that he forgot $P(x+3)$. Suppose that $P(x) = 17, P(x+1) = 44, P(x+2) = 50, P(x+4) = 5$. If $P(x+3)$ can be written as $\frac{m}{n}$ for positive integers m, n, find $m+n$.

Problem 7

Suppose that for positive reals x and y: $x+y = 10$. If the minimum value of $(1+\frac{1}{x})(1+\frac{1}{y})$ can be written as $\frac{m}{n}$ where $gcd(m,n) = 1$, find $m+n$.

Problem 8

Find the smallest positive integer n such that $n^4 - 8n^3 + 24n^2 - 32n + 665$ is a perfect square.

Problem 9

Define

$$f(a,b,c) = lcm(\gcd(a,b), lcm(a,c), bc)$$

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Given that prime factorization $f(2022, 9009, 7)$ can be written as $p_1^{e_1} .. p_k^{e_k}$, find $p_1 e_1 + ... + p_k e_k$.

Problem 10

How many ordered septuples (a, b, c, d, e, f, g) of nonnegative integers satisfy $a^4 + b^4 + c^4 + d^4 + e^4 + f^4 + 7 = 40^g$?